# $CP$ Violation and the Light Quark Sector [*]

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## Abstract

In view of the observed strong hierarchy of quark masses, we discuss a new description of flavor mixing which is particularly suited for models of quark mass matrices based on flavor symmetries. The necessary and sufficient conditions for $CP$ violation are clarified. The emergence of $CP$ violation is primarily linked to a large phase difference (near $90°$) in the light quark sector. The unitarity triangle is determined by the mass ratios of the light quarks. We conclude that the unitarity triangle should be close or identical to a rectangular triangle, and $CP$ violation is maximal in this sense.

arXiv:hep-ph/9909303v1  9 Sep 1999

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[*]Invited talk given at the 13th Rencontres de Physique de la Vallee d'Aoste, *Results and Perspectives in Particle Physics*, La Thuile, Aosta Valley, Italy, March 1999

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# 1    Introduction

A deeper understanding of flavor mixing and $CP$ violation, observed in the weak interactions, remains one of the major challenges in particle physics. In the standard electroweak theory with three quark families the phenomenon of flavor mixing is described by a $3 \times 3$ unitary matrix, which can be expressed in terms of four independent parameters, usually taken to be three rotation angles and one complex phase. There seems no way to obtain any further information about these parameters within the standard model. Any attempt to do so would require new physical inputs which are beyond the standard model.

At the present time it seems hopeless to find a complete solution to the fermion mass and flavor mixing problem by theoretical insight alone. One can hope, however, to detect a specific order in the tower of fermion masses and the four parameters of quark flavor mixing, especially in observing links between the parameters of the flavor mixing and the mass eigenvalues. That such links should exist, seems obvious to us. Like in any quantum mechanical system the mixing pattern of the states will influence the pattern of the mass eigenvalues, and vice versa. One possible way to make these links more transparent is to look for specific symmetry limits, e.g., by setting parameters, which are observed to be small, to zero and to study the situation in the symmetry limit first. Following such an approach, we shall demonstrate that (a) a specific description of quark flavor mixing can be derived, (b) two of the three flavor mixing angles are related directly to the quark mass ratios $m_u/m_c$ and $m_d/m_s$, and (c) the unitarity triangle of quark mixing related to $CP$ violation in $B$-meson decays is fixed in terms of these mass ratios and the modulus of the Cabibbo transition element $|V_{us}|$. Furthermore we shall give arguments why an inner angle of the unitarity triangle (angle $\alpha$) should be equal to 90° or close to 90° [1].

The "standard" parametrization of the flavor mixing matrix (advocated by the Particle Data Group [2]) and the original Kobayashi-Maskawa parametrization [3] were introduced without taking possible links between the quark masses and the flavor mixing parameters into account. The parametrization introduced some time ago [4, 5] is based on such a connection, although the specific relations between flavor mixing angles and quark masses might be more complicated than commonly envisaged. It is a parametrization which allows to interpret the phenomenon of flavor mixing as an evolutionary or tumbling process. In the limit in which the masses of the light quarks $(u, d)$ and the medially light quarks $(c, s)$ are set to zero, while the heavy quarks $(t, b)$ acquire their masses, there is no flavor mixing [6]. Once the masses of the $(c, s)$ quarks are introduced, while the $(u, d)$ quarks remain massless, the flavor mixing is reduced to an admixture between two families, described by one angle $\theta$. As soon as the $u$- and $d$-quark masses are introduced as small perturbations, the full flavor mixing matrix involving a complex phase parameter and two more mixing angles $(\theta_u, \theta_d)$ appears. These angles can be interpreted as rotations between the states $(u, c)$ and $(d, s)$, respectively. In either the "standard" parametrization or the Kobayashi-Maskawa representation, however, such specific limits are difficult to consider. For this reason we proceed to describe the flavor mixing by use of the parametrization given in Ref. [4].

# 2 The flavor mixing matrix

In the standard electroweak theory or those extensions which have no flavor-changing right-handed currents, it is always possible to choose a basis of flavor space in which the up- and down-type quark mass matrices are hermitian. Without loss of any generality the (1,3) and (3,1) elements of both mass matrices can further be arranged, through a common unitary transformation, to be zero [4]. Then one is left with hermitian quark mass matrices of the form

$$
M_{\rm q} = \begin{pmatrix} E_{\rm q} & D_{\rm q} & \mathbf{0} \\ D_{\rm q}^* & C_{\rm q} & B_{\rm q} \\ \mathbf{0} & B_{\rm q}^* & A_{\rm q} \end{pmatrix} ,
\tag{2.1}
$$

where q $=$ u (up) or d (down), and the hierarchy $|A_{\rm q}| \gg |B_{\rm q}|, |C_{\rm q}| \gg |D_{\rm q}|, |E_{\rm q}|$ is generally expected. In this basis, there is no direct mixing between the heavy $t$ (or $b$) quark and the light $u$ (or $d$) quark in $M_{\rm u}$ (or $M_{\rm d}$), i.e., the quark mass matrix is close to the well-known form of "nearest-neighbour" interactions [7].

A mass matrix of the type (2.1) can in the absence of complex phases be diagonalized by a $3 \times 3$ orthogonal matrix, described only by two rotation angles in the hierarchy limit of quark masses [8]. First, the off-diagonal element $B_{\rm q}$ is rotated away by a rotation matrix $R_{23}$ between the second and third families. Then the element $D_{\rm q}$ is rotated away by a transformation $R_{12}$ between the first and second families. No rotation between the first and third families is necessary in either the limit $m_u \to 0$, $m_d \to 0$ or the limit $m_t \to \infty$, $m_b \to \infty$. Lifting such a hierarchy limit, which is not far from the reality, one needs an additional transformation $R_{31}$ with a tiny rotation angle to fully diagonalize $M_{\rm q}$. Note, however, that the rotation sequence $(R_{12}^{\rm u} R_{23}^{\rm u})(R_{12}^{\rm d} R_{23}^{\rm d})^{\rm T}$ is enough to describe the $3 \times 3$ real flavor mixing matrix, as the effects of $R_{31}^{\rm u}$ and $R_{31}^{\rm d}$ can always be absorbed into this sequence through redefining the relevant rotation angles. By introducing a complex phase angle into the rotation combination $(R_{23}^{\rm u})(R_{23}^{\rm d})^{\rm T}$, we finally arrive at the following representation of quark flavor mixing [4]:

$$
\begin{aligned}
V &= \begin{pmatrix} c_{\rm u} & s_{\rm u} & 0 \\ -s_{\rm u} & c_{\rm u} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_{\rm d} & -s_{\rm d} & 0 \\ s_{\rm d} & c_{\rm d} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} s_{\rm u} s_{\rm d} c + c_{\rm u} c_{\rm d} e^{-i\varphi} & s_{\rm u} c_{\rm d} c - c_{\rm u} s_{\rm d} e^{-i\varphi} & s_{\rm u} s \\ c_{\rm u} s_{\rm d} c - s_{\rm u} c_{\rm d} e^{-i\varphi} & c_{\rm u} c_{\rm d} c + s_{\rm u} s_{\rm d} e^{-i\varphi} & c_{\rm u} s \\ -s_{\rm d} s & -c_{\rm d} s & c \end{pmatrix} ,
\end{aligned}
\tag{2.2}
$$

where $s_{\rm u} \equiv \sin\theta_{\rm u}$, $c_{\rm u} \equiv \cos\theta_{\rm u}$, etc. The three mixing angles can all be arranged to lie in the first quadrant, i.e., all $s_{\rm u}$, $s_{\rm d}$, $s$ and $c_{\rm u}$, $c_{\rm d}$, $c$ are positive. The phase $\varphi$ may in general take all values between 0 and $2\pi$. Clearly $CP$ violation is present, if $\varphi \neq 0$ or $\pi$.

Although we have derived in a heuristic way the particular description of the flavor mixing matrix (2.2) from the hierarchical mass matrix (2.1), we should like to emphasize that (2.2) is a possible way to describe any mixing matrix, one out of nine inequivalent representations classified in Ref. [5].

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If the phase $\varphi$ in $V$ is disregarded, the resulting rotation matrix (obtained from (2.2) for $\varphi = 0$) is just the one used originally by Euler; i.e., the angles $\theta$, $\theta_u$ and $\theta_d$ correspond to the usual Euler angles [9]. Note that this is not the case for other representations of the flavor mixing matrix given in the literature [3, 10]. The representation given in (2.2) can be interpreted as follows. First, a rotation by the angle $\theta_d$ takes place in the plane defined by the $d$ and $s$ quarks. It is followed by a rotation (angle $\theta$) in the $b$–$s'$ plane, where $s'$ denotes the superposition $s' = d \sin \theta_d + s \cos \theta_d$. At the same time the orthogonal state $d' = d \cos \theta_d - s \sin \theta_d$ is multiplied by the phase factor $e^{-i\varphi}$. Finally a rotation (angle $\theta_u$) is applied in the 1–2 plane (about the new third axis).

The sequence of rotations corresponds just to the Euler sequence [9]: $R_{12} R_{23} R_{12}^{\mathrm{T}}$. On the other hand, the original Kobayashi-Maskawa representation [3] corresponds to the sequence $R_{23} R_{12} R_{23}^{\mathrm{T}}$, while the "standard" representation [2] corresponds to the sequence $R_{23} R_{31} R_{12}$ (see also the classifications given in Ref. [5]). Although all descriptions of the flavor mixing matrix are mathematically equivalent, we emphasize that the Euler sequence $R_{12} R_{23} R_{12}^{\mathrm{T}}$ is physically of particular interest, as it involves the rotation matrices $R_{12}$ and $R_{12}^{\mathrm{T}}$, which describe the rotations in the light quark sector, in a symmetric way. Since the flavor mixing matrix acts between the quark mass eigenstates $\mathcal{U} = (u, c, t)$ and $\mathcal{D} = (d, s, b)$, one could absorb the two $R_{12}$ rotations in a redefinition of the quark fields. The charged weak transition term can be rewritten as follows:

$$\overline{\mathcal{U}}_{\mathrm{L}} \, V \, \mathcal{D}_{\mathrm{L}} = \overline{(u, \ c, \ t)}_{\mathrm{L}} \, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\mathrm{L}} = \overline{(u', \ c', \ t)}_{\mathrm{L}} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} d' \\ s' \\ b \end{pmatrix}_{\mathrm{L}} , \qquad (2.3)$$

where $u' = u \cos \theta_u - c \sin \theta_u$ and $c' = c \cos \theta_u + u \sin \theta_u$. Thus the angles $\theta_u$ and $\theta_d$ describe the corresponding rotations in the $(u, c)$ and $(d, s)$ systems.

We should like to emphasize that the angles $\theta_u$ and $\theta_d$ can directly be measured from weak decays of $B$ mesons and from $B^0$-$\bar{B}^0$ mixing. An analysis of the present experimental data yields [11]: $\theta_u = 4.87° \pm 0.86°$ and $\theta_d = 11.71° \pm 1.09°$. Due to the symmetric structure of our mixing matrix (2.2), we are able to interpret the $\theta_d$ and $\theta_u$ rotations as specific transformations of the corresponding mass eigenstates. Such an interpretation is not possible for the third rotation given by $\theta$, measured to be $2.30° \pm 0.09°$ [11]. This rotation takes place between the third family of the massive quarks and the $c'$ and $s'$ states. One interpretation would be to associate the rotation of $\theta$ with a transformation among $b$ and $s'$. Another possibility is to describe the effect as a rotation among $t$ and $c'$. However, one could also write $\theta$ as a difference of two other angles, and describe the mixing effect as a combination of a rotation in the $(b, s')$ system and a rotation in the $(t, c')$ system. Thus a unique interpretation does not exist. We remark that the asymmetry between the $\theta$ rotation on the one hand and the $\theta_u$ and $\theta_d$ rotations on the other hand is a direct consequence of our flavor mixing matrix (which is in turn related to the hierarchical structure of the mass spectrum) and is primarily linked to the fact that there exist three different quark families.

As summarized in Refs. [4, 5], the new parametrization (2.2) has a number of advantages over all the others in the study of heavy flavor decays and quark mass matrices. Its usefulness

will be seen more clearly in the present work. As an example we explore the interesting connection between our parametrization (2.2) and the unitarity triangle of quark mixing defined by the orthogonality relation

$$V_{ub}^* V_{ud} \ + \ V_{cb}^* V_{cd} \ + \ V_{tb}^* V_{td} \ = \ 0 \tag{2.4}$$

in the complex plane. The inner angles of this triangle, usually denoted as

$$\alpha \ = \ \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right) \ ,$$

$$\beta \ = \ \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right) \ ,$$

$$\gamma \ = \ \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right) \ , \tag{2.5}$$

can be determined from some $CP$ asymmetries in $B$-meson decays [12]. The parametrization (2.2) takes an instructive leading-order form:

$$V \ \approx \ \begin{pmatrix} e^{-\mathrm{i}\alpha} & s_{\mathrm{C}} e^{\mathrm{i}\gamma} & s_{\mathrm{u}} s \\ s_{\mathrm{C}} e^{\mathrm{i}\beta} & 1 & s \\ -s_{\mathrm{d}} s & -s & 1 \end{pmatrix} \ , \tag{2.6}$$

where $s_{\mathrm{C}} \equiv \sin\theta_{\mathrm{C}} \approx |s_{\mathrm{u}} - s_{\mathrm{d}} e^{-\mathrm{i}\varphi}|$ with $\theta_{\mathrm{C}}$ denoting the Cabibbo rotation angle [13]. Clearly $\alpha \approx \varphi$ holds as a straightforward result of (2.6). In this approximation $|V_{ub}^* V_{ud}|$, $|V_{cb}^* V_{cd}|$ and $|V_{tb}^* V_{td}|$, the three sides of the unitarity triangle (2.4), are rescaled to $s_{\mathrm{u}}$, $s_{\mathrm{d}}$ and $s_{\mathrm{C}}$ respectively. The latter are three sides of a new triangle with smaller area ($\approx s_{\mathrm{u}} s_{\mathrm{d}} \sin\alpha / 2$), which will subsequently be referred to as the "light-quark triangle" in the heavy quark limit ($m_t \to \infty$, $m_b \to \infty$). The values of $\alpha$, $\beta$ and $\gamma$ can therefore be given in terms of $s_{\mathrm{u}}$, $s_{\mathrm{d}}$ and $s_{\mathrm{C}}$ with the help of the cosine theorem. In particular, relations like [4]

$$\sin\alpha \ : \ \sin\beta \ : \ \sin\gamma \ \approx \ s_{\mathrm{C}} \ : \ s_{\mathrm{u}} \ : \ s_{\mathrm{d}} \tag{2.7}$$

may directly be confronted with the upcoming data on $CP$ asymmetries in $B$ decays [14]. Motivated by these interesting results, we shall investigate the role that the light quark sector plays in $CP$ violation for a variety of realistic textures of quark mass matrices.

## 3  Symmetry limits

We remark two useful limits of quark masses and analyze their corresponding consequences on flavor mixing. In the limit $m_u \to 0$, $m_d \to 0$ ("chiral limit"), where both the up and down quark mass matrices have zeros in the positions $(1,1)$, $(1,2)$, $(2,1)$, $(1,3)$ and $(3,1)$ (see also Ref. [6]), the flavor mixing angles $\theta_{\mathrm{u}}$ and $\theta_{\mathrm{d}}$ vanish. Only the $\theta$ rotation affecting the heavy quark sector remains, i.e., the flavor mixing matrix effectively takes the form

$$\hat{V} \ = \ \begin{pmatrix} \cos\hat{\theta} & \sin\hat{\theta} \\ -\sin\hat{\theta} & \cos\hat{\theta} \end{pmatrix} \ , \tag{3.1}$$

where $\hat{\theta}$ denotes the value of $\theta$ which one obtains in the limit $\theta_u \to 0$, $\theta_d \to 0$. We see that $\hat{V}$ is a real orthogonal matrix, arising naturally from $V$ in the chiral limit.

The flavor mixing angle $\hat{\theta}$ can be derived from hermitian quark mass matrices of the following general form (in the limit $m_u \to 0$, $m_d \to 0$):

$$\hat{M}_q = \begin{pmatrix} \hat{C}_q & \hat{B}_q \\ \hat{B}_q^* & \hat{A}_q \end{pmatrix} , \tag{3.2}$$

where $|\hat{A}_q| \gg |\hat{B}_q|, |\hat{C}_q|$; and q = u (up) or d (down). Note that the phase difference between $\hat{B}_u$ and $\hat{B}_d$, denoted as $\kappa \equiv \arg(\hat{B}_u) - \arg(\hat{B}_d)$, has no effect on $CP$ symmetry in the chiral limit, but it may affect the magnitude of $\hat{\theta}$. It is known that current data on the top-quark mass and the $B$-meson lifetime disfavor the special case $\hat{C}_u = \hat{C}_d = 0$ for $\hat{M}_u$ and $\hat{M}_d$ (see, e.g., Ref. [15]), hence we take $\hat{C}_q \neq 0$ and define a ratio $\hat{r}_q \equiv |\hat{B}_q|/\hat{C}_q$. We can obtain the flavor mixing angle $\hat{\theta}$, in terms of the quark mass ratios $m_c/m_t$, $m_s/m_b$ and the parameters $\hat{r}_u$, $\hat{r}_d$, by diagonalizing the mass matrices in (3.2). In the next-to-leading order approximation, $\sin\hat{\theta}$ reads

$$\sin\hat{\theta} = \left| \hat{r}_d \frac{m_s}{m_b} \left(1 - \hat{\delta}_d\right) - \hat{r}_u \frac{m_c}{m_t} \left(1 - \hat{\delta}_u\right) e^{i\kappa} \right| , \tag{3.3}$$

where two correction terms are given by

$$\hat{\delta}_u = \left(1 + \hat{r}_u^2\right) \frac{m_c}{m_t} ,$$
$$\hat{\delta}_d = \left(1 + \hat{r}_d^2\right) \frac{m_s}{m_b} . \tag{3.4}$$

In view of the fact $m_s/m_b \sim O(10)\, m_c/m_t$ from current data [2, 16], we find that the flavor mixing angle $\hat{\theta}$ is primarily linked to $m_s/m_b$ provided $|\hat{r}_u| \approx |\hat{r}_d|$. Note that in specific models, e.g., those describing the mixing between the second and third families as an effect related to the breaking of an underlying "democratic symmetry" [17, 18], the ratios $\hat{r}_u$ and $\hat{r}_d$ are purely algebraic numbers (such as $|\hat{r}_u| = |\hat{r}_d| = 1/\sqrt{2}$ or $\sqrt{2}$).

For illustration, we take $\hat{r}_u = \hat{r}_d \equiv \hat{r}$ to fit the experimental result $\sin\hat{\theta} = 0.040 \pm 0.002$ with the typical inputs $m_b/m_s = 26 - 36$ and $m_t/m_c \sim 250$. It is found that the favored value of $|\hat{r}|$ varies in the range 1.0 – 2.5, dependent weakly on the phase parameter $\kappa$.

Note that both $m_s/m_b$ and $m_c/m_t$ evolve with the energy scale (e.g., from the weak scale $\mu \sim 10^2$ GeV to a superhigh scale $\mu \sim 10^{16}$ GeV, or vice versa), therefore $\tilde{\theta}$ is a scale-dependent quantity.

The limit $m_t \to \infty$, $m_b \to \infty$ is subsequently referred to as the "heavy quark limit". In this limit, in which the $(3,3)$ elements of the up and down mass matrices formally approach infinity but all other matrix elements are fixed, the angle $\theta$ vanishes. The flavor mixing matrix, which is nontrivial only in the light quark sector, takes the form:

$$\tilde{V} = \begin{pmatrix} \tilde{c}_u & \tilde{s}_u \\ -\tilde{s}_u & \tilde{c}_u \end{pmatrix} \begin{pmatrix} e^{-i\tilde{\varphi}} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{c}_d & -\tilde{s}_d \\ \tilde{s}_d & \tilde{c}_d \end{pmatrix}$$
$$= \begin{pmatrix} \tilde{s}_u \tilde{s}_d + \tilde{c}_u \tilde{c}_d e^{-i\tilde{\varphi}} & \tilde{s}_u \tilde{c}_d - \tilde{c}_u \tilde{s}_d e^{-i\tilde{\varphi}} \\ \tilde{c}_u \tilde{s}_d - \tilde{s}_u \tilde{c}_d e^{-i\tilde{\varphi}} & \tilde{c}_u \tilde{c}_d + \tilde{s}_u \tilde{s}_d e^{-i\tilde{\varphi}} \end{pmatrix} . \tag{3.5}$$

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where $\tilde{s}_u = \sin\tilde{\theta}_u$, $\tilde{c}_u = \cos\tilde{\theta}_u$, etc. The angles $\tilde{\theta}_u$ and $\tilde{\theta}_d$ are the values for $\theta_u$ and $\theta_d$ obtained in the heavy quark limit. Since the $(t, b)$ system is decoupled from the $(c, s)$ and $(u, d)$ systems, the flavor mixing can be described as in the case of two families. Therefore the mixing matrix $\tilde{V}$ is effectively given in terms of only a single rotation angle, the Cabbibo angle $\theta_C$:

$$\sin\theta_C = | \; \tilde{s}_u\tilde{c}_d \; - \; \tilde{c}_u\tilde{s}_d \; e^{-i\tilde{\varphi}} \; | \; . \tag{3.6}$$

Of course $\tilde{V}(\theta_C)$ is essentially a real matrix, because its complex phases can always be rotated away by redefining the quark fields.

We should like to stress that the heavy quark limit, which carries the flavor mixing matrix $V$ to its simplified form $\tilde{V}$, is not far from the reality, since $1-c \approx 0.1\%$ holds [4]. Therefore $\theta_u$, $\theta_d$ and $\varphi$ are expected to approach $\tilde{\theta}_u$, $\tilde{\theta}_d$ and $\tilde{\varphi}$ rapidly, as $\theta \to 0$, corresponding to $m_t \to \infty$ and $m_b \to \infty$. However, the concrete limiting behavior depends on the specific algebraic structure of the up and down mass matrices. If two hermitian mass matrices have the parallel hierarchy with texture zeros in the (1,1) (2,2), (1,3) and (3,1) elements, for example, the magnitude of $\theta$ is suppressed by the terms proportional to $m_t^{-1/2}$ and $m_b^{-1/2}$ [8]; and if the (2,2) elements are kept nonvanishing and comparable in magnitude with the (2,3) and (3,2) elements, then $\theta$ is dependent on $m_t^{-1}$ and $m_b^{-1}$ [17, 18].

The angles $\tilde{\theta}_u$ and $\tilde{\theta}_d$ as well as the phase $\tilde{\varphi}$ are well-defined quantities in the heavy quark limit. The physical meaning of these quantities can be seen more clearly, if we take into account a specific and realistic model for the Cabibbo-type mixing in the light quark sector. It is well known that in the absence of the $u$-quark mass a relation between the Cabibbo angle $\theta_C$ and the mass ration $m_d/m_s$ follows, if the quark mass matrices have the structure:

$$\tilde{M}_u = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_c \end{pmatrix},$$
$$\tilde{M}_d = \begin{pmatrix} \mathbf{0} & \tilde{B}_d \\ \tilde{B}_d^* & \tilde{A}_d \end{pmatrix}. \tag{3.7}$$

The diagonalization of $\tilde{M}_d$ leads to the relation $\tan\theta_C = \sqrt{m_d/m_s}$ . The texture-zero pattern of $\tilde{M}_d$, i.e., the vanishing of its (1,1) element, is already present in certain classes of models (see, e.g., Refs. [19, 20]). The relation for the Cabibbo angle is known to agree very well with the experimental observation. For numerical discussions, we make use of the quark masses $m_u = (5.1 \pm 0.9)$ MeV, $m_d = (9.3 \pm 1.4)$ MeV, $m_s = (175 \pm 25)$ MeV and $m_c = (1.35 \pm 0.05)$ GeV at the scale $\mu = 1$ GeV [16]. Then one finds $\theta_C = 13.0° \pm 1.8°$ or $\sin\theta_C = 0.225 \pm 0.031$, consistent with the observed value of $|V_{us}|$ (i.e., $0.217 \le |V_{us}| \le 0.224$ [2]).

The situation will change once $m_u$ is introduced, i.e., $\tilde{M}_u$ takes the same form as $\tilde{M}_d$ given in (3.7). In this case the mass matrices result in the following relation [8]:

$$\sin\theta_C = | \; R_u \; - \; R_d \; e^{-i\psi} \; | \; , \tag{3.8}$$

where

$$R_u = \sqrt{\frac{m_u}{m_u + m_c}} \sqrt{\frac{m_s}{m_d + m_s}} \; ,$$
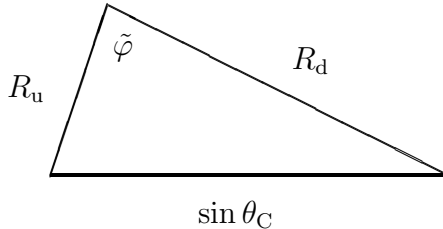
Figure 1: The light-quark triangle (LT) in the complex plane.

$$R_{\mathrm{d}} = \sqrt{\frac{m_c}{m_u + m_c}} \sqrt{\frac{m_d}{m_d + m_s}} \ , \tag{3.9}$$

and $\psi \equiv \arg(\tilde{B}_{\mathrm{u}}) - \arg(\tilde{B}_{\mathrm{d}})$ denotes the relative phase between the off-diagonal elements $\tilde{B}_{\mathrm{u}}$ and $\tilde{B}_{\mathrm{d}}$ (in the limit $m_u \to 0$ this phase can be absorbed through a redifinition of the quark fields). We find that the same structure for the Cabibbo-type mixing matrix has been obtained as in the decoupling limit discussed above. If we set

$$
\begin{aligned}
\tan \tilde{\theta}_{\mathrm{u}} &= \sqrt{\frac{m_u}{m_c}} \ , \\
\tan \tilde{\theta}_{\mathrm{d}} &= \sqrt{\frac{m_d}{m_s}} \ ,
\end{aligned}
\tag{3.10}
$$

and $\tilde{\varphi} = \psi$ for (3.6), then the result in (3.8) and (3.9) can exactly be reproduced.

Indeed the relation in (3.6) or (3.8) defines a triangle in the complex plane, which will be denoted as the "light-quark triangle"(LT). Taking into account the central values of the Cabibbo angle ($\sin \theta_{\mathrm{C}} = |V_{us}| = 0.2205$) and the light quark mass ratios ($m_s/m_d = 18.8$ and $m_c/m_u = 265$), we can calculate the phase parameter from (3.8) and obtain $\tilde{\varphi} = \psi \approx 79°$. If we allow the mass ratios and $\theta_{\mathrm{C}}$ to vary in their ranges given above, then $\tilde{\varphi}$ may vary in the range $38° - 115°$. We find that $\tilde{\varphi}$ has a good chance to be $90°$ (see also Ref. [15]). The case $\tilde{\varphi} \approx 90°$ (i.e., the LT is rectangular) is of special interest, since it implies that the area of the unitarity triangle of flavor mixing takes its maximum value for the fixed quark mass ratios – in this sense, the $CP$ symmetry of weak interactions would be maximally violated.

The two symmetry limits discussed above are both not far from the reality, in which the strong hierarchy of quark masses ($m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$) has been observed. They will serve as a guide in the subsequent discussions about generic quark mass matrices and their consequences on flavor mixing.

# 4   The texture of mass matrices

We return to the case of three quark families. We adopt a basis of flavor space in which both the up- and down-type quark mass matrices are hermitian and have vanishing (1,3) and (3,1) elements, as shown in (2.1). Such a basis is of special interest in case of a strong mass hierarchy (as realized by nature), since no explicit mixing between the very massive $t$ (or $b$)

quark and the very light $u$ (or $d$) quark is introduced. The mixing can then be regarded as of the "nearest neighbour" type. Thus without loss of generality one may discuss the model-independent properties of flavor mixing and $CP$ violation on the basis of the mass matrices (2.1), i.e.,

$$
\begin{aligned}
M_{\mathrm{u}} &= \begin{pmatrix} E_{\mathrm{u}} & D_{\mathrm{u}} & \mathbf{0} \\ D_{\mathrm{u}}^* & C_{\mathrm{u}} & B_{\mathrm{u}} \\ \mathbf{0} & B_{\mathrm{u}}^* & A_{\mathrm{u}} \end{pmatrix} , \\
M_{\mathrm{d}} &= \begin{pmatrix} E_{\mathrm{d}} & D_{\mathrm{d}} & \mathbf{0} \\ D_{\mathrm{d}}^* & C_{\mathrm{d}} & B_{\mathrm{d}} \\ \mathbf{0} & B_{\mathrm{d}}^* & A_{\mathrm{d}} \end{pmatrix} .
\end{aligned}
\tag{4.1}
$$

The phases of $D_{\mathrm{u,d}}$ and $B_{\mathrm{u,d}}$ elements are denoted as $\phi_{D_{\mathrm{u,d}}}$ and $\phi_{B_{\mathrm{u,d}}}$, respectively. The phase differences

$$
\begin{aligned}
\phi_1 &= \phi_{D_{\mathrm{u}}} - \phi_{D_{\mathrm{d}}} , \\
\phi_2 &= \phi_{B_{\mathrm{u}}} - \phi_{B_{\mathrm{d}}}
\end{aligned}
\tag{4.2}
$$

are the source of $CP$ violation in weak interactions of quarks. It is clear that $M_{\mathrm{u}}$ and $M_{\mathrm{d}}$ consist totally of twelve parameters.

This general case has been discussed in Ref. [1], where it is pointed out that the observations indicate that both $E_{\mathrm{u}}$ and $E_{\mathrm{d}}$ elements are either very small or zero. We proceed to specify the general hermitian mass matrices by taking $E_{\mathrm{q}} = 0$:

$$
M_{\mathrm{q}} = \begin{pmatrix} \mathbf{0} & D_{\mathrm{q}} & \mathbf{0} \\ D_{\mathrm{q}}^* & C_{\mathrm{q}} & B_{\mathrm{q}} \\ \mathbf{0} & B_{\mathrm{q}}^* & A_{\mathrm{q}} \end{pmatrix} .
\tag{4.3}
$$

In case of two quark families, this is just the form taken for $\tilde{M}_{\mathrm{d}}$ in (3.7). As remarked above, the texture zeros in (1,3) and (3,1) positions can always be arranged. Thus the physical constraint is as follows: in the flavor basis in which (1,3) and (3,1) elements of $M_{\mathrm{u,d}}$ vanish, the (1,1) element of $M_{\mathrm{u,d}}$ vanishes as well. This can strictly be true only at a particular energy scale. The vanishing of the (1,1) element can be viewed as a result of an underlying flavor symmetry, which may either be discrete or continuous.

In the literature a number of such possibilities have been discussed (see, e.g., Refs. [8] – [21]). Here we shall not discuss further details in this respect, but concentrate on the phenomenological consequences of such a texture pattern. It is particularly interesting that some predictions of this ansatz for the mixing angles and the unitarity triangle are approximately independent of the renormalization-group effects, therefore a specification of the energy scale at which the texture of $M_{\mathrm{u,d}}$ holds is unnecessary for our purpose. We believe that $M_{\mathrm{q}}$ given in (4.3) is a realistic candidate for the quark mass matrices of a (yet unknown) fundamental theory responsible for fermion mass generation and $CP$ violation, and we shall make some further speculations about this point at the end of this talk.

We define $|B_q|/C_q \equiv r_q$ for each quark sector. The magnitude of $r_q$ is expected to be of $O(1)$. The parameters $A_q$, $|B_q|$, $C_q$ and $|D_q|$ in (4.3) can be expressed in terms of the quark mass eigenvalues and $r_q$. We obtain the three mixing angles of $V$ as follows:

$$
\begin{aligned}
\tan\theta_u &= \sqrt{\frac{m_u}{m_c}}\,(1+\Delta_u)\;, \\
\tan\theta_d &= \sqrt{\frac{m_d}{m_s}}\,(1+\Delta_d)\;, \\
\sin\theta &= \left| r_d \frac{m_s}{m_b}(1-\delta_d) \;-\; r_u \frac{m_c}{m_t}(1-\delta_u)\, e^{i\phi_2} \right|\;,
\end{aligned}
\tag{4.4}
$$

where the next-to-leading order corrections read

$$
\begin{aligned}
\Delta_u &= \sqrt{\frac{m_c m_d}{m_u m_s}}\,\frac{m_s}{m_b}\left| \mathrm{Re}\left[ e^{i\phi_1} - \frac{r_u}{r_d}\cdot\frac{m_c m_b}{m_t m_s}e^{i(\phi_1+\phi_2)} \right]^{-1} \right|\;, \\
\Delta_d &= \sqrt{\frac{m_u m_s}{m_c m_d}}\,\frac{m_c}{m_t}\left| \mathrm{Re}\left[ e^{i\phi_1} - \frac{r_d}{r_u}\cdot\frac{m_t m_s}{m_c m_b}e^{i(\phi_1+\phi_2)} \right]^{-1} \right|\;;
\end{aligned}
\tag{4.5}
$$

and

$$
\begin{aligned}
\delta_u &= \frac{m_u}{m_c} + \left(1+r_u^2\right)\frac{m_c}{m_t}\;, \\
\delta_d &= \frac{m_d}{m_s} + \left(1+r_d^2\right)\frac{m_s}{m_b}\;.
\end{aligned}
\tag{4.6}
$$

Clearly the result for $\hat{\delta}_{u,d}$ in (3.4) can be reproduced from $\delta_{u,d}$ in (4.6), if one takes the chiral limit $m_u \to 0$, $m_d \to 0$. We also observe that the phase $\phi_2$ is only associated with the small quantity $m_c/m_t$ in $\sin\theta$. To get the relationship between $\varphi$ and $\phi_1$ or $\phi_2$, we first calculate $|V_{us}|$:

$$
|V_{us}| = \left(1 - \frac{1}{2}\frac{m_u}{m_c} - \frac{1}{2}\frac{m_d}{m_s}\right)\left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\,e^{i\phi_1} \right|
\tag{4.7}
$$

in the next-to-leading order approximation. Note that this result can also be achieved from (3.8) and (3.9), which were obtained in the heavy quark limit. Confronting (4.7) with current data on $|V_{us}|$ leads to the result $\phi_1 \sim 90°$, as we have discussed before. Therefore $\cos\phi_1$ is expected to be a small quantity. Then we use (4.20) together with (4.4) and (4.7) to calculate $\cos\varphi$. In the same order approximation, we arrive at

$$
\cos\varphi = \sqrt{\frac{m_u m_s}{m_c m_d}}\,\Delta_u + \sqrt{\frac{m_c m_d}{m_u m_s}}\,\Delta_d + (1-\Delta_u-\Delta_d)\cos\phi_1\;.
\tag{4.8}
$$

The contribution of $\phi_2$ to $\varphi$ is substantially suppressed at this level of accuracy.

For simplicity, we proceed by taking $r_u = r_d \equiv r$, which holds in some models with natural flavor symmetries [17]. Then $\sin\theta$ becomes proportional to a universal parameter $|r|$. In view of the fact $m_s/m_b \sim O(10)\, m_c/m_t$, we find that the result in (4.5) can be simplified as

$$
\begin{aligned}
\Delta_u &= \sqrt{\frac{m_c m_d}{m_u m_s}}\,\frac{m_s}{m_b}\,\cos\phi_1\;, \\
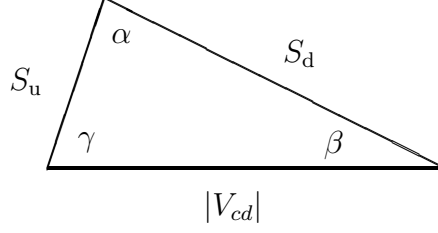\Delta_d &= 0\;.
\end{aligned}
\tag{4.9}
$$

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Figure 2: The rescaled unitarity triangle (UT) in the complex plane.

Also the relation between $\varphi$ and $\phi_1$ in (4.8) is simplified to

$$\cos\varphi = \left(1 + \frac{m_s}{m_b}\right)\cos\phi_1 . \qquad (4.10)$$

As $m_s/m_b \sim 4\%$, it becomes apparent that $\varphi \approx \phi_1$ is a good approximation. Note that $\phi_1 = \varphi$ holds exactly in the heavy quark limit, in which $\varphi$ has been denoted as $\tilde{\varphi}$ (see (3.5) as well as Fig. 1). The equality $\phi_1 = \tilde{\varphi}$ follows, i.e., both stand for the phase difference between the mass matrix elements $D_u$ and $D_d$.

Let us calculate the parameter $\mathcal{J}$ $(= s_u c_u s_d c_d s^2 c \sin\varphi$ [4]), a rephasing-invariant quantity of $CP$ violation in the quark sector. We find that the magnitude of $\mathcal{J}$ is dominated by the $\sin\phi_1$ term and receives one-order smaller corrections from the $\sin(\phi_1 \pm \phi_2)$ terms. As a result,

$$\mathcal{J} \approx |r|^2 \sqrt{\frac{m_u}{m_c}}\sqrt{\frac{m_d}{m_s}}\left(\frac{m_s}{m_b}\right)^2 \sin\phi_1 \qquad (4.11)$$

holds to a good degree of accuracy. Clearly $\mathcal{J} \sim O(10^{-5}) \times \sin\phi_1$ with $\sin\phi_1 \sim 1$ is favored by current data.

The result of $\mathcal{J}$ in (4.12) might give the impression that $CP$ violation is absent if either $m_u$ or $m_d$ vanishes. This is not exactly true, however. If we set $m_u = 0$, $\mathcal{J}$ is not zero, but it becomes dependent on $\sin\phi_2$ with a factor which is about two orders of magnitude smaller (i.e., of order $10^{-7}$):

$$\mathcal{J} \approx |r|^2 \frac{m_c}{m_t} \cdot \frac{m_d}{m_s}\left(\frac{m_s}{m_b}\right)^2 \sin\phi_2 . \qquad (4.12)$$

Certainly this possibility is already ruled out by experimental data.

Note also that the model predicts

$$\tan\theta_u = \left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}}\,(1 + \Delta_u) ,$$

$$\tan\theta_d = \left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{m_d}{m_s}}\,(1 + \Delta_d) , \qquad (4.13)$$

a result obtained first by one of us from a more specific pattern of quark mass matrices [7]. In $B$-meson physics, $|V_{ub}/V_{cb}|$ can be determined from the ratio of the decay rate of $B \to (\pi, \rho)l\nu_l$ to that of $B \to D^* l\nu_l$ ; and $|V_{td}/V_{ts}|$ can be extracted from the ratio of the rate of $B_d^0$-$\bar{B}_d^0$ mixing to that of $B_s^0$-$\bar{B}_s^0$ mixing.

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We are now in a position to calculate the unitarity triangle (UT) of quark flavor mixing defined in (2.8), whose three inner angles are denoted as $\alpha$, $\beta$ and $\gamma$ in (2.9). Note that three sides of the unitarity triangle can be rescaled by $V_{cb}^*$ (see Fig. 2 for illustration). The resultant triangle reads

$$|V_{cd}| = \left| S_{\mathrm{d}} - S_{\mathrm{u}}\, e^{-i\alpha} \right| , \qquad (4.14)$$

where $S_{\mathrm{u}} = |V_{ub}V_{ud}/V_{cb}|$ and $S_{\mathrm{d}} = |V_{tb}V_{td}/V_{cb}|$. After some calculations $S_{\mathrm{u}}$, $S_{\mathrm{d}}$ and $\alpha$ are obtained from the above quark mass texture in the next-to-leading order approximation:

$$
\begin{aligned}
S_{\mathrm{u}} &= \sqrt{\frac{m_u}{m_c}} \left( 1 - \frac{1}{2}\frac{m_u}{m_c} - \frac{1}{2}\frac{m_d}{m_s} + \sqrt{\frac{m_c m_d}{m_u m_s}}\,\frac{m_s}{m_b}\,\cos\phi_1 + \sqrt{\frac{m_u m_d}{m_c m_s}}\,\cos\phi_1 \right) , \\
S_{\mathrm{d}} &= \sqrt{\frac{m_d}{m_s}} \left( 1 + \frac{1}{2}\frac{m_u}{m_c} - \frac{1}{2}\frac{m_d}{m_s} \right) ;
\end{aligned}
\qquad (4.15)
$$

and

$$\sin\alpha = \left( 1 - \sqrt{\frac{m_u m_d}{m_c m_s}}\,\cos\phi_1 \right) \sin\phi_1 . \qquad (4.16)$$

A comparison of the rescaled UT in Fig. 2 with the LT in Fig. 1, which is obtained in the heavy quark limit, is interesting. We find

$$
\begin{aligned}
\frac{S_{\mathrm{u}} - R_{\mathrm{u}}}{R_{\mathrm{u}}} &= \left( 1 + \frac{m_c m_s}{m_u m_b} \right) \sqrt{\frac{m_u m_d}{m_c m_s}}\,\cos\tilde{\varphi} , \\
\frac{S_{\mathrm{d}} - R_{\mathrm{d}}}{R_{\mathrm{d}}} &= \frac{m_u}{m_c} , \\
\frac{\sin\alpha - \sin\tilde{\varphi}}{\sin\tilde{\varphi}} &= -\sqrt{\frac{m_u m_d}{m_c m_s}}\,\cos\tilde{\varphi} ,
\end{aligned}
\qquad (4.17)
$$

which are of order $15\%\cos\tilde{\varphi}$, $0.4\%$ and $1.4\%\cos\tilde{\varphi}$, respectively. Obviously $R_{\mathrm{d}} \approx S_{\mathrm{d}}$ is an excellent approximation, and $\alpha \approx \tilde{\varphi} \approx \varphi$ is a good approximation. As $\varphi$ (or $\tilde{\varphi}$) is expected to be close to $90°$, $R_{\mathrm{u}} \approx S_{\mathrm{u}}$ should also be accurate enough in the next-to-leading order estimation. Therefore the light-quark triangle is essentially *congruent with* the rescaled unitarity triangle! This result has two straightforward implications: first, $CP$ violation is an effect arising primarily from the light quark sector; second, the $CP$-violating observables ($\alpha$, $\beta$, $\gamma$) can be predicted in terms of the light quark masses and the phase difference between up and down mass matrices [15]. If we use the value of $|V_{cd}|$, which is expected to equal $|V_{us}|$ within the $0.1\%$ error bar, then all three angles of the unitarity triangle can be calculated in terms of $m_u/m_c$, $m_d/m_s$ and $|V_{cd}|$ to a good degree of accuracy.

The three angles of the UT ($\alpha$, $\beta$ and $\gamma$) will be well determined at the $B$-meson factories, e.g., from the $CP$ asymmetries in $B_d \to \pi^+\pi^-$, $B_d \to J/\psi K_S$ and $B_u^\pm \to (D^0, \bar{D}^0) + K^{(*)\pm}$ decays [12]. The characteristic measurable quantities are $\sin(2\alpha)$, $\sin(2\beta)$ and $\sin^2\gamma$, respectively. For the purpose of illustration, we typically take $|V_{us}| = |V_{cd}| = 0.22$, $m_u/m_c = 0.0056$, $m_d/m_s = 0.045$ and $m_s/m_b = 0.033$ to calculate these three $CP$-violating parameters from the LT and from the rescaled UT separately. Both approaches lead to $\alpha \approx 90°$, $\beta \approx 20°$ and $\gamma \approx 70°$, which are in good agreement with the results obtained from the standard analysis of current data on $|V_{ub}/V_{cb}|$, $\epsilon_K$, $B_d^0$-$\bar{B}_d^0$ mixing and $B_s^0$-$\bar{B}_s^0$ mixing [11]. Note that among three

$CP$-violating observables only $\sin(2\beta)$ is remarkably sensitive to the value of $m_u/m_c$, which involves quite large uncertainty (e.g., $\sin(2\beta)$ may change from 0.4 to 0.8 if $m_u/m_c$ varies in the range $0.002 - 0.01$). For this reason we emphasize again that the numbers given above can only serve as an illustration. A more reliable determination of the quark mass values is crucial, in order to test the ansätze of quark mass matrices in a numerically decisive way

It is also worth mentioning that the result $\tan\theta_{\rm d} = \sqrt{m_d/m_s}$ is particularly interesting for the mixing rates of $B_d^0$-$\bar{B}_d^0$ and $B_s^0$-$\bar{B}_s^0$ systems, measured by $x_{\rm d}$ and $x_{\rm s}$ respectively [2]. The ratio $x_{\rm s}/x_{\rm d}$ amounts to $|V_{ts}/V_{td}|^2 = \tan^{-2}\theta_{\rm d}$ multiplied by a factor $\chi_{\rm su(3)} = 1.45 \pm 0.13$, which reflects the $\text{SU}(3)_{\rm flavor}$ symmetry breaking effects. As $x_{\rm d} = 0.723 \pm 0.032$ has been well determined [2], the prediction for the value of $x_{\rm s}$ is

$$x_{\rm s} \;=\; x_{\rm d}\; \chi_{\rm su(3)}\; \frac{m_s}{m_d} \;=\; 19.8 \pm 3.5\;, \tag{4.18}$$

where $m_s/m_d = 18.9 \pm 0.8$, obtained from the chiral perturbation theory [16], has been used. This result is certainly consistent with the present experimental bound on $x_{\rm s}$, i.e., $x_{\rm s} > 14.0$ at the 95% confidence level [2]. A measurement of $x_{\rm s} \sim 20$ may be realized at the forthcoming HERA-$B$ and LHC-$B$ experiments.

# 5 Discussions and conclusion

We have studied the phenomena of quark flavor mixing and $CP$ violation in the context of generic hermitian mass matrices. The necessary and sufficient conditions for $CP$ violation in the standard model have been clarified at both the level of quark mass matrices and that of the flavor mixing matrix. Our particular observation is that $CP$ violation is primarily linked to a phase difference of about $90°$ in the light quark sector, and this property becomes most apparent in the new parametrization (2.2). To be more specific, we have analyzed a realistic pattern of quark mass matrices with four texture zeros and given predictions for the flavor mixing and $CP$-violating parameters. The approximate congruency between the light-quark triangle (LT) and the rescaled unitarity triangle (UT), which provides an intuitive and scale-independent connection of $CP$-violating observables to quark mass ratios, is particularly worth mentioning.

Let us make some further comments on the quark mass matrix (4.3), its phenomenological hints and its theoretical prospects.

Naively one might not expect any prediction from the four-texture-zero mass matrices in (4.3), since they totally consist of ten free parameters (two of them are the phase differences between $M_{\rm u}$ and $M_{\rm d}$). This is not true, however, as we have seen. We find that two predictions, $\tan\theta_{\rm u} \approx \sqrt{m_u/m_c}$ and $\tan\theta_{\rm d} \approx \sqrt{m_d/m_s}$, can be obtained in the leading order approximation. In some cases the latter may even hold in the next-to-leading order approximation, as shown in (4.4) and (4.9). Note again that these two relations, as a consequence of the hierarchy and texture zeros of our quark mass matrices, are essentially independent of the renormalization-

group effects. This interesting scale-independent feature can also be seen from the LT and the rescaled UT as well as their inner angles $(\alpha, \beta, \gamma)$.

It remains to be seen whether the interesting possibility $\varphi \approx \phi_1 \approx 90°$, indicated by current data of quark masses and flavor mixing, could arise from an underlying flavor symmetry or a dynamical symmetry breaking scheme. Some speculations about this problem have been made (see, e.g., Refs. [15] and Refs. [17, 18]). However, no final conclusion has been reached thus far. It is remarkable, nevertheless, that we have at least observed a useful relation between the area of the UT ($\mathcal{A}_{\mathrm{UT}}$) and that of the LT ($\mathcal{A}_{\mathrm{LT}}$) to a good degree of accuracy:

$$\mathcal{A}_{\mathrm{UT}} \approx |V_{cb}|^2 \mathcal{A}_{\mathrm{LT}} \approx \sin^2\theta \ \mathcal{A}_{\mathrm{LT}} . \tag{5.1}$$

Since $\mathcal{A}_{\mathrm{UT}} = \mathcal{J}/2$ measures the magnitude of $CP$ violation in the standard model, we conclude that $CP$ violation is primarily linked to the light quark sector. This is a natural consequence of the strong hierarchy between the heavy and light quark masses, which is on the other hand responsible for the smallness of $\mathcal{J}$ or $\mathcal{A}_{\mathrm{UT}}$.

Is it possible to derive the quark mass matrix (4.3) in some theoretical frameworks? To answer this question we first specify the hierarchical structure of $M_{\mathrm{q}}$ in terms of the mixing angle $\theta_{\mathrm{q}}$ (for q = d or s). Adopting the radiant unit for the mixing angles (i.e., $\theta_{\mathrm{u}} \approx 0.085$, $\theta_{\mathrm{d}} \approx 0.204$ and $\theta \approx 0.040$), we have

$$\begin{aligned}
\frac{m_u}{m_c} &\sim \frac{m_c}{m_t} \sim \theta_{\mathrm{u}}^2 , \\
\frac{m_d}{m_s} &\sim \frac{m_s}{m_b} \sim \theta_{\mathrm{d}}^2 .
\end{aligned} \tag{5.2}$$

Then the mass matrices $M_{\mathrm{u}}$ and $M_{\mathrm{d}}$, which have the mass scales $m_t$ and $m_b$ respectively, take the following *parallel* hierarchies:

$$M_{\mathrm{u}} \sim m_t \begin{pmatrix} \mathbf{0} & \theta_{\mathrm{u}}^3 & \mathbf{0} \\ \theta_{\mathrm{u}}^3 & \theta_{\mathrm{u}}^2 & \theta_{\mathrm{u}}^2 \\ \mathbf{0} & \theta_{\mathrm{u}}^2 & \mathbf{1} \end{pmatrix} ,$$

$$M_{\mathrm{d}} \sim m_b \begin{pmatrix} \mathbf{0} & \theta_{\mathrm{d}}^3 & \mathbf{0} \\ \theta_{\mathrm{d}}^3 & \theta_{\mathrm{d}}^2 & \theta_{\mathrm{d}}^2 \\ \mathbf{0} & \theta_{\mathrm{d}}^2 & \mathbf{1} \end{pmatrix} , \tag{5.3}$$

where the relevant complex phases have been neglected. Clearly all three flavor mixing angles can properly be reproduced from (5.3), once one takes $\theta \approx \theta_{\mathrm{d}}^2 \gg \theta_{\mathrm{u}}^2$ into account. The $CP$-violating phase $\varphi$ in $V$ comes essentially from the phase difference between the $\theta_{\mathrm{u}}^3$ and $\theta_{\mathrm{d}}^3$ terms.

Of course $\theta_{\mathrm{u}}$ and $\theta_{\mathrm{d}}$, which are more fundamental than the Cabibbo angle $\theta_{\mathrm{C}}$ in our point of view, denote perturbative corrections to the rank-one limits of $M_{\mathrm{u}}$ and $M_{\mathrm{d}}$ respectively. They are responsible for the generation of light quark masses as well as the flavor mixing. They might also be responsible for $CP$ violation in a specific theoretical framework (e.g., the pure real $\theta_{\mathrm{u}}$ and the pure imaginary $\theta_{\mathrm{d}}$ might lead to a phase difference of about $90°$ between $M_{\mathrm{u}}$ and $M_{\mathrm{d}}$, which is just the source of $CP$ violation favored by current data). The

small parameter $\theta_q$ could get its physical meaning in the Yukawa coupling of an underlying superstring theory: $\theta_q = \langle\Theta_q\rangle/\Omega_q$, where $\langle\Theta_q\rangle$ denotes the vacuum expectation value of the singlet field $\Theta_q$, and $\Omega_q$ represents the unification (or string) mass scale which governs higher dimension operators (see, e.g., Ref. [22]). The quark mass matrices of the form (5.3) could then be obtained by introducing an extra (horizontal) U(1) gauge symmetry or assigning the matter fields appropriately.

A detailed study of possible dynamical models responsible for the quark mass matrices (4.3) or (5.3) is certainly desirable. We believe that the texture zeros and parallel hierarchies of up and down quark mass matrices do imply specific symmetries, perhaps at a superhigh scale, and have instructive consequences on flavor mixing and $CP$-violating phenomena. The new parametrization of the flavor mixing matrix that we advocated is particularly useful in studying the quark mass generation, flavor mixing and $CP$ violation.

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